A perception-referenced method for comparison of radiance ratio spectra and its application as an index of metamerism

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ABSTRACT

A metric for comparison of radiance ratio (e.g., reflectance) spectra, based on colorimetric principles, is described. In essence, the metric is a linear approximation to the sum of a series of $\Delta E^*_{ab}$ values wherein the two spectra differ only within a single narrow wavelength band. This metric has previously been suggested as a measure of lack-of-fit between a spectral-based color model and experimental observations, as well as an optimization criterion in modeling the color behavior of color output devices.

In this paper, the application of the metric as an index of metamerism is presented. Unlike the current CIE-recommended special metameric indices, the new proposal does not require the specification of a single set of trial conditions. Further, unlike previous spectrum-based proposals, it provides results in familiar units of $\Delta E^*_{ab}$.

Keywords: metamerism, general index of metamerism, metameric index, spectral comparison index.

1. INTRODUCTION

Color stimuli are termed metameric if they match in color under a set of reference conditions, such as the 1931 Standard Observer and Illuminant D65, but possess radiance ratio spectra which are different. A metameric index quantifies, in some sense, the extent to which the two spectra differ.

Current CIE recommendations for evaluating metamerism include the special metameric indices for change in illuminant [1] and change in observer. [2] These consist of, in essence, the total color difference ($\Delta E$ or $\Delta E^*$) between the two stimuli under a set of conditions which differ from the reference conditions under which the two stimuli match in color. The change in conditions is limited to either a change in illuminant (in the case of the special metameric index for change in illuminant) or change in observer (for the special metameric index for change in observer).

While there are some instances in which a single change in test conditions is of interest, there are many more for which several test conditions bear on the acceptability of a metameric match. For example, goods which are purchased under cool white fluorescent lighting, for example, would be expected to match also under other types of fluorescent lighting, incandescent lighting, various daylight conditions, and as viewed by a number of observers.

Naturally, in such situations, the special metameric indices may be computed for a variety of observer/illuminant test conditions, and the results presented in tabular form. In order to arrive at a single-valued result, the maximum of the tabulated values may be taken and reported.

There have been instances where, in assessing the extent to which a predicted (as by a model such as Kubelka-Munk) spectrum differs from the actual spectrum, researchers have used some norm of the difference between the radiance ratio spectra [3] or the log radiance (density) spectra. [4] Another suggestion [5] was based upon the correlation coefficient between the two spectra. These techniques provide a single number for each spectrum pair, but, because they afford equal weight to all wavelengths (the first is also radiometrically linear), they cannot be regarded as being referenced to the human visual system.

Nimeroff and Yurow [6] developed a spectral-based metameric index which was based upon a weighted sum of the absolute differences between the two radiance ratio spectra. The weights were computed based on the color matching functions used in the 1964 U* V* W* uniform color space, and depended upon the tristimulus values of the stimuli under the reference conditions. Thus, a different set of weights was used for each metameric pair. Unfortunately, this
metric does not report results in familiar units, and, while an improvement over unweighted norms of the difference spectrum, is not perceptually uniform.

It is highly desirable, then, to possess a metamerism index which is perceptually-referenced, produces results in familiar units of $\Delta E^*_{ab}$ (or a more modern version), and provides a single-valued result which is the maximum difference to be encountered under practical conditions.

2. A PERCEPTION-REFERENCED METHOD FOR COMPARISON OF RADIANCE RATIO SPECTRA

A method has been proposed by this author for the comparison of two radiance ratio spectra which are reported at regular wavelength intervals. [7] In concept, the method is based upon the sum across the wavelengths of the $\Delta E^*$ values between the two spectra, wherein they differ at only that particular wavelength. In practice, a linearized approximation to these $\Delta E^*$s is used. The result is a weighted sum of the absolute values of the differences between the two spectra.

The Spectral Comparison Index is computed as:

$$M_v = \sum \lambda w(\lambda) \cdot |\Delta \beta(\lambda)|$$  \hspace{1cm} (1)

where $\Delta \beta(\lambda)$ is the difference between the two radiance ratio spectra, and

$$w(\lambda) = \sqrt{\left(\frac{dL^*}{d\beta(\lambda)}\right)^2 + \left(\frac{da^*}{d\beta(\lambda)}\right)^2 + \left(\frac{db^*}{d\beta(\lambda)}\right)^2}$$  \hspace{1cm} (2)

The derivatives of $L^*$, $a^*$, and $b^*$ with respect to $\beta(\lambda)$ are computed via the chain rule:

$$\frac{dL^*}{d\beta(\lambda)} = 116 \cdot k \cdot s(\lambda) \cdot \frac{d}{dY} f\left(\frac{Y}{Y_n}\right)$$

$$\frac{da^*}{d\beta(\lambda)} = 500 \cdot k \cdot s(\lambda) \cdot \left[\frac{d}{dX} f\left(\frac{X}{X_n}\right) - \frac{\tilde{Y}(\lambda)}{\bar{Y}(\lambda)} \cdot \frac{d}{dY} f\left(\frac{Y}{Y_n}\right)\right]$$

$$\frac{db^*}{d\beta(\lambda)} = 200 \cdot k \cdot s(\lambda) \cdot \left[\frac{d}{dZ} f\left(\frac{Z}{Z_n}\right) - \frac{\tilde{Z}(\lambda)}{\bar{Z}(\lambda)} \cdot \frac{d}{dZ} f\left(\frac{Z}{Z_n}\right)\right]$$  \hspace{1cm} (3)

and, further:

$$\frac{d}{du} f\left(\frac{u}{u_n}\right) = \begin{cases} 7.787, & u \leq 0.008856 \\ \frac{1}{3} f\left(\frac{u}{u_n}\right), & u > 0.008856 \end{cases}$$  \hspace{1cm} (4)

where $u$ is replaced by, in turn, $X$, $Y$, and $Z$, and $u_n$ by the corresponding tristimulus value of the specified white object.

Table 1 contains the weights for three stimuli, by way of example. The first is for a medium-dark neutral, with an $L^*$ of 30. The second is for a light neutral, with an $L^*$ of 80. Notice how the weights for the lighter neutral are always smaller than those for the darker. The ratio, greater than 4, is precisely the ratio of the luminances of the two stimuli raised to
the power of $-2/3$. This illustrates the dependence of the weights on the overall darkness or lightness of the stimulus. (The relationship is close to that proposed by Nimeroff and Yurow, but slightly refined.) The third set of weights are for a Yellow stimulus, with the same $L^*$ and $a^*$ as the light gray. Note that the weights for the Yellow stimulus are larger than those for the light gray stimulus below 500 nm. This illustrates the dependence of the weights upon the color of the stimulus.

Figure 1 contains the data from Table 1 in graphic form.

Table 1. The weights for three stimuli, for the reference conditions 1931 Standard Observer, Illuminant D65.

<table>
<thead>
<tr>
<th>$L^*$:</th>
<th>30</th>
<th>80</th>
<th>80</th>
<th>$w_l$, nm</th>
<th>$w_1$</th>
<th>$w_2$</th>
<th>$w_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a^*$:</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>540</td>
<td>82.170</td>
<td>18.866</td>
<td>18.741</td>
</tr>
<tr>
<td>$b^*$:</td>
<td>0</td>
<td>0</td>
<td>60</td>
<td>550</td>
<td>73.411</td>
<td>16.855</td>
<td>16.793</td>
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</table>

<table>
<thead>
<tr>
<th>$wl$, nm</th>
<th>$w_1$</th>
<th>$w_2$</th>
<th>$w_3$</th>
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<tr>
<td>400</td>
<td>2.779</td>
<td>0.638</td>
<td>1.394</td>
</tr>
<tr>
<td>410</td>
<td>8.036</td>
<td>1.845</td>
<td>4.034</td>
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<td>420</td>
<td>25.816</td>
<td>5.927</td>
<td>12.995</td>
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<tr>
<td>430</td>
<td>51.926</td>
<td>11.922</td>
<td>26.306</td>
</tr>
<tr>
<td>440</td>
<td>74.887</td>
<td>17.194</td>
<td>38.391</td>
</tr>
<tr>
<td>450</td>
<td>82.826</td>
<td>19.017</td>
<td>43.335</td>
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<tr>
<td>460</td>
<td>75.115</td>
<td>17.246</td>
<td>40.641</td>
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<td>470</td>
<td>52.604</td>
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<td>480</td>
<td>28.659</td>
<td>6.580</td>
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<td>490</td>
<td>22.193</td>
<td>5.096</td>
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<td>8.296</td>
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<td>510</td>
<td>56.895</td>
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</tr>
<tr>
<td>520</td>
<td>75.079</td>
<td>17.238</td>
<td>16.895</td>
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<tr>
<td>530</td>
<td>84.474</td>
<td>19.395</td>
<td>19.173</td>
</tr>
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</table>

3. APPLICATION AS AN INDEX OF METAMERISM

When computing a metameric index, one is, in a manner of speaking, comparing the extent to which two spectra are different. In fact, there have been some instances in which investigators have utilized a CIE special metameric index as
an indication of how visually different two spectra are. [3] The relationship between the two applications of assessment of metamerism and difference between two spectra are closely related.

We propose that the \( M_v \) index, in Equation (1) above, be used as a metameric index. It has the advantage of being firmly rooted in visual perception, and is based upon the familiar units of \( \Delta E_{ab}^* \). Further, our practical experience with this index suggests that the \( M_v \) index represents an upper bound for all special metameric indices computable under practical conditions.

### 3.1 Parameric Decomposition
If the test spectrum is a strict metamer of the standard (tristimulus values equal), the formula given above may be used directly. If, on the other hand, the test spectrum is a paramer (tristimulus values approximately equal), the method proposed by Fairman [8] may be used to compute the difference spectrum to be used in Equation (1).

### 4. CONCLUSIONS
A general index of metamerism has been presented. The index is based upon a linearized approximation to a CIELAB color difference computed on a wavelength-by-wavelength basis. Unlike special indices of metamerism, it does not depend upon one or more test conditions (i.e., observer/illuminant combination).

Like the index proposed by Nimeroff and Yurow, the new index is the weighted sum of the absolute difference between the standard and a trial spectrum. Also in agreement with Nimeroff and Yurow, the weights for the new index are computed using a set of color matching functions, and are higher for darker standard stimuli and smaller for lighter standards. Unlike this previous index, the new index provides results in familiar units of \( \Delta E_{ab}^* \).

### LITERATURE CITED


5. ISCC Project Committee 27, Indices of Metamerism. Undated typescript.

